Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/25**

Further Mathematics
Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. Two particles, P and Q, of masses $3m$ and $2m$ respectively, are moving on a smooth horizontal plane. They are moving in opposite directions along the same straight line when they collide directly.

Immediately before the collision, P is moving with speed $2u$.

The magnitude of the impulse exerted on P by Q in the collision is $\dfrac{9mu}{2}$

(a) Find the speed of P immediately after the collision.

(3)

The coefficient of restitution between P and Q is e.

Given that the speed of Q immediately before the collision is u,

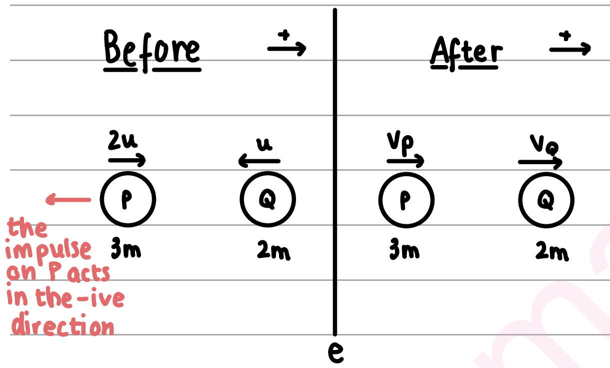
(b) find the value of e.

(5)

(a) Impulse is the change in momentum M1

Formula for change in momentum:

$$\Delta\text{momentum} = \overset{\text{mass}}{m}v_{\text{final}} - m\underset{\text{velocity}}{v_{\text{initial}}}$$



Substitute:

we're looking for v_P

A1 $-\dfrac{9\cancel{m}u}{2} = 3\cancel{m}v_P - 3\cancel{m}(2u)$ cancel m's

Impulse on P is in negative direction

$-\dfrac{9}{2}u + 6u = 3v_P$

$\dfrac{3}{2}u = v_P$

$\boxed{\dfrac{1}{2}u = v_P}$ speed of P after the impulse A1

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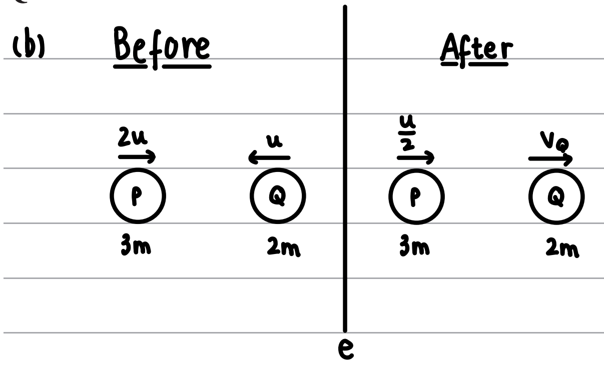
Question 1 continued

(b)



We can use the conservation of linear momentum to get v_Q first. M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A \underbrace{u_A}_{\text{initial velocity}} + m_B u_B = m_A \underbrace{v_A}_{\text{final velocity}} + m_B v_B$

Substitute: $3\cancel{m}(2u) + 2\cancel{m}(-u) = 3\cancel{m}\left(\frac{u}{2}\right) + 2\cancel{m}v_Q$ A1

$$6u - 2u = \frac{3}{2}u + 2v_Q$$

$$4u - \frac{3}{2}u = 2v_Q$$

$$\frac{2.5u}{2} = v_Q \longrightarrow v_Q = \frac{5}{4}u$$

We can use Newton's Law of Restitution to get the value of e. M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $\underbrace{e}_{\text{coefficient of restitution}}(\underbrace{u_A}_{\text{initial speed}} - u_B) = v_B - \underbrace{v_A}_{\text{final speed}}$

Substitute: $e(2u - -u) = \frac{5}{4}u - \frac{u}{2}$ A1

$$3e\cancel{u} = \frac{3}{4}\cancel{u} \longrightarrow \boxed{e = \frac{1}{4} \quad \text{value of } e}$$ A1

(Total for Question 1 is 8 marks)

2. A racing car of mass 750 kg is moving along a straight horizontal road at a constant speed of U **km h**$^{-1}$. The engine of the racing car is working at a constant rate of 60 kW.

The resistance to the motion of the racing car is modelled as a force of magnitude $37.5v$ N, where v m s^{-1} is the speed of the racing car.

Using the model,

(a) find the value of U

(4)

Later on, the racing car is accelerating up a straight road which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{5}{49}$. The engine of the racing car is working at a constant rate of 60 kW.

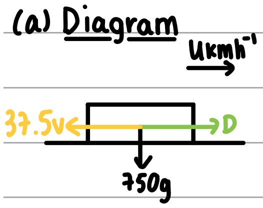
The total resistance to the motion of the racing car from non-gravitational forces is modelled as a force of magnitude $37.5v$ N, where v m s^{-1} is the speed of the racing car. At the instant when the acceleration of the racing car is 2 m s^{-2}, the speed of the racing car is V m s^{-1}

Using the model,

(b) find the value of V

(4)

(a) Diagram



The speed is constant $\therefore$ use $\Sigma F_x = 0$

$D = 37.5(u)$ M1

To get D we will use Power.

Formula for Power:

Power (W) — $P = Dv$

Driving force (N)   velocity (ms⁻¹)

$P = 60\text{kW} \xrightarrow{\times 1000} 60\,000\text{W}$
$v = u\ \text{kmh}^{-1}$
$D = D$

Substitute:

$60\,000 = u \times D$

$D = \frac{60\,000}{u}$ M1

Substitute our D into the equation above:

$\frac{60\,000}{u} = 37.5u$ A1

$60\,000 = \frac{75}{2}u^2$

$u^2 = 1600$

$u = 40\text{ms}^{-1}$

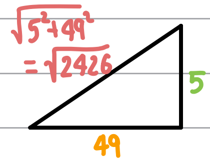
$\downarrow$ ÷1000, ×3600 to convert to kmh⁻¹

144kmh^{-1} A1

P 7 2 8 1 1 A 0 4 1 6

Question 2 continued

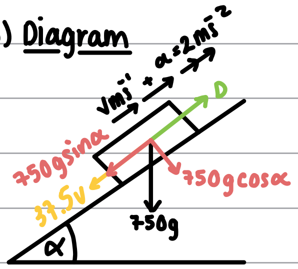
$\tan\alpha = \frac{5}{49}$

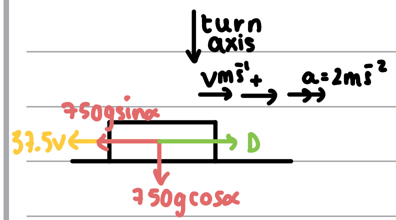


$\therefore \cos\alpha = \frac{49}{\sqrt{2426}}$

$\sin\alpha = \frac{5}{\sqrt{2426}}$

(b) Diagram





Since it's accelerating, use $\Sigma F_x = ma$:

$D - 37.5v - 750g\sin\alpha = 750(2)$ M1A1

To get D we will use Power.

Formula for Power:

$\underset{\text{Power (W)}}{P} = \underset{\text{Driving force (N)}}{D}\,\underset{\text{velocity (ms}^{-1}\text{)}}{v}$

$P = 60\text{kW} \xrightarrow{\times 1000} 60\,000\text{W}$

$v = v$

$D = D$

Substitute:

$60\,000 = Dv$

$D = \frac{60\,000}{v}$ ← substitute this back

$\frac{60\,000}{v} - 37.5v - 750g\sin\alpha = 1500$ A1

$0 = 37.5v^2 + (1500 + 750g\sin\alpha)v - 60\,000$

Use Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$v = \frac{-(1500 + 750g\sin\alpha) + \sqrt{(1500 + 750g\sin\alpha)^2 - 4(37.5)(-60\,000)}}{2(37.5)}$

$v = 20\,ms^{-1}$ value of v A1

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. A stone of mass 0.5 kg is projected vertically upwards with a speed $U\,\text{m s}^{-1}$ from a point A. The point A is 2.5 m above horizontal ground.

The speed of the stone as it hits the ground is $25\,\text{m s}^{-1}$

The motion of the stone from the instant it is projected from A until the instant it hits the ground is modelled as that of a particle moving freely under gravity.

(a) Use the model and the principle of conservation of mechanical energy to find the value of U.

(4)

In reality, the stone will be subject to air resistance as it moves from A to the ground.

(b) State how this would affect your answer to part (a).

(1)

The ground is soft and the stone sinks a vertical distance d cm into the ground. The resistive force exerted on the stone by the ground is modelled as a constant force of magnitude 2000 N and the stone is modelled as a particle.

(c) Use the model and the work-energy principle to find the value of d, giving your answer to 3 significant figures.

(5)

★ conservation of mechanical energy principle: states that the total amount of mechanical energy (KE/GPE) in a closed system in the absence of dissipative forces (e.g. friction/air resistance) remains constant. M1

★ Remember the mechanical energy formulae:

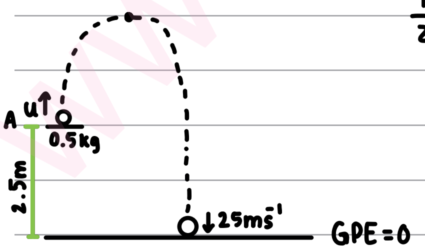
$$\underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} = \underbrace{KE_f}_{\text{final kinetic}} + \underbrace{GPE_f}_{\text{final grav. potential}}$$

★ Formulae for KE and GPE:

$$KE = \frac{1}{2}mv^2 \quad (m \text{ mass}, v \text{ velocity})$$

$$GPE = mgh \quad (m \text{ mass}, h \text{ change in height}, g = 9.8\,\text{ms}^{-2})$$

(a) Diagram



★ We need to set GPE=0 so we can have a reference point.

Substitute:

$$\frac{1}{2}\left(\frac{1}{2}\right)(u)^2 + \frac{1}{2}g(2.5) = \frac{1}{2}\left(\frac{1}{2}\right)(25)^2 + \cancel{\tfrac{1}{2}g(0)}^{\,0} \quad \text{A1A1}$$

$$\frac{1}{4}u^2 + \frac{5}{4}g = \frac{625}{4}$$

$$u^2 = 625 - 5g$$

$$u = \sqrt{576}$$

$$u = 24 \quad \text{A1}$$

Question 3 continued

(b) The value of u would be larger B1

(c) ★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction). M1

★ Remember the work-energy formulae:

Either: $\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} = \underbrace{KE_f}_{\text{final kinetic}} + \overbrace{GPE_f}^{\text{final grav. potential}} + \underbrace{\text{WD against friction}}_{\text{work lost to friction}}$

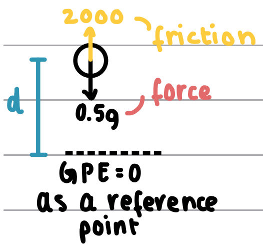
OR: $\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} - \underbrace{\text{WD by friction}}_{\text{we subtract this since it leaves the system as heat!}} = \underbrace{KE_f}_{\text{final kinetic}} + \overbrace{GPE_f}^{\text{final grav. potential}}$

★ Formulae for KE and GPE:

$KE = \frac{1}{2}mv^2$ (m: mass, v: velocity)

$GPE = mgh$ (m: mass, h: change in height, $g = 9.8\,ms^{-2}$)

Diagram



Substitute:

$(0.5g)d + \frac{1}{2}(0.5)(25)^2 + 0.5gd - 2000d = \cancel{\frac{1}{2}(0.5)(0)^2}^{\,0} + \cancel{0.5g(0)}^{\,0}$

$\frac{9.8}{2}d + \frac{625}{4} + \frac{9.8d}{2} - 2000d = 0$ M1A1

$\frac{625}{4} = 2000d - 9.8d$ — d is in cm. we want d in meters $\therefore \frac{d}{100} = 0.01d$

A1 $\frac{625}{4} = 0.01 \times 2000d - 0.01(9.8)d$

$\frac{625}{4} = 20d - 0.098d$

$\boxed{d = 7.83 \text{ to } 3sf}$ A1

Question 3 continued

P 7 2 8 1 1 A 0 1 0 1 6

Question 3 continued

(Total for Question 3 is 10 marks)

4.

Figure 1

Three particles, P, Q and R, lie at rest on a smooth horizontal plane. The particles are in a straight line with Q between P and R, as shown in Figure 1.

Particle P is projected **towards** Q with speed u. At the same time, R is projected with speed $\frac{1}{2}u$ **away from** Q, in the direction QR.

Particle P has mass m and particle Q has mass $2m$.

The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision between P and Q is $\dfrac{u(1+e)}{3}$

(6)

It is given that $e > \frac{1}{2}$

(b) Determine whether there is a collision between Q and R.

(2)

(c) Determine the direction of motion of P immediately after the collision between P and Q.

(2)

(d) Find, in terms of m, u and e, the total kinetic energy lost in the collision between P and Q, simplifying your answer.

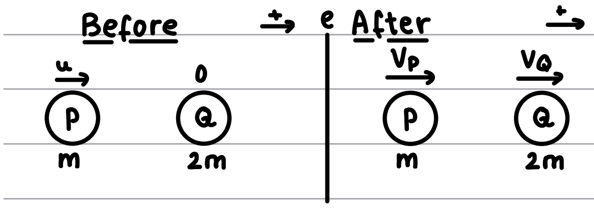
(3)

(e) Explain how using $e = 1$ could be used to check your answer to part (d).

(1)

Question 4 continued

(a) Diagram (P and Q)



We can use the conservation of linear momentum to get this. M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A V_A + m_B V_B$

u_A initial velocity, V_A final velocity

Substitute:

$$\cancel{m}(u) + 2\cancel{m}(0) = \cancel{m}V_P + 2\cancel{m}V_Q \quad \text{cancel } m\text{'s}$$

$$u = V_P + 2V_Q \quad \text{Eq1} \quad \text{A1}$$

We can use Newton's Law of Restitution to get an equation. M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = V_B - V_A$

coefficient of restitution (e), initial speed (u_A), final speed (V_A)

Substitute:

$$e(u-0) = V_Q - V_P$$

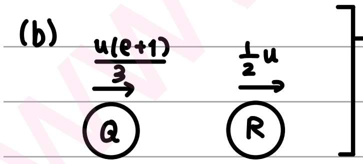
$$eu = V_Q - V_P \quad \text{Eq2} \quad \text{A1}$$

Solve simultaneously Eq1 and Eq2:

$$u = V_P + 2V_Q$$
$$eu = V_Q - V_P \quad + \text{ use elimination method}$$
$$eu + u = 3V_Q$$

$$\frac{u}{3}(e+1) = V_Q \quad \text{hence shown} \quad \text{M1A1}$$

(b)



Compare speeds: for them to collide, $V_Q > V_R$

$$\frac{\cancel{u}}{3}(e+1) > \frac{1}{2}\cancel{u}$$

$$\frac{1}{3}(e+1) > \frac{1}{2}$$

if $e > \frac{1}{2}$, $(e+1) > \frac{3}{2}$

$$\therefore \frac{u(1+e)}{3} > \frac{1}{2}u \quad \therefore \text{there's a collision between Q and R}$$

M1A1

Question 4 continued

(c) Get V_P:

$$V_Q = \frac{u(e+1)}{3} \longrightarrow u = V_P + 2V_Q \rightarrow u = V_P + 2\left(\frac{u(e+1)}{3}\right)$$

$$u - \frac{2}{3}u(e+1) = V_P$$

$$u\left(1 - \frac{2}{3}e - \frac{2}{3}\right) = V_P$$

$$u\left(\frac{1}{3} - \frac{2}{3}e\right) = V_P$$

$$V_P = \frac{u(1-2e)}{3} \quad \text{M1}$$

$e > \frac{1}{2}$ $\therefore V_P < 0$ $(\xrightarrow{+})$ $\therefore$ P changes direction A1

(d) To get KE lost: $\Delta KE = KE_I - KE_F$

Formula for Kinetic Energy:

$$KE = \frac{1}{2}mv^2$$

mass velocity

Substitute: $\Delta KE = \frac{1}{2}mu^2 - \left(\frac{1}{2}m\left(\frac{u(1-2e)}{3}\right)^2 + \frac{1}{\cancel{2}}(\cancel{2}m)\left(\frac{u(e+1)}{3}\right)^2\right)$ M1A1

expand $= \frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u^2}{9}(1-4e+4e^2)\right) - m\left(\frac{u^2}{9}(e^2+2e+1)\right)$

collect like terms $= \frac{1}{2}mu^2 - \frac{mu^2}{18} + \frac{4emu^2}{18} - \frac{4mu^2e^2}{18} - \frac{mu^2e^2}{9} - \frac{2emu^2}{9} - \frac{mu^2}{9}$

$$= \frac{9}{18}mu^2 - \frac{mu^2}{18} - \frac{2mu^2}{18} + \cancel{\frac{4emu^2}{18}} - \cancel{\frac{4emu^2}{18}} - \frac{6mu^2e^2}{18}$$

$$= \frac{\cancel{6}^{1}}{\cancel{18}_{3}}mu^2 - \frac{\cancel{6}^{1}}{\cancel{18}_{3}}mu^2e^2$$

$$= \frac{1}{3}mu^2 - \frac{1}{3}mu^2e^2 \text{ — factorize} \rightarrow \frac{1}{3}mu^2[1-e^2] = \Delta KE \quad \text{A1}$$

(e) When $e=1$, the answer to (d) will be 0. ($e=1$ is a perfectly elastic collision, no KE lost) B1

Question 4 continued

Question 4 continued

(Total for Question 4 is 14 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS